10.4 Use Inscribed Angles and Polygons

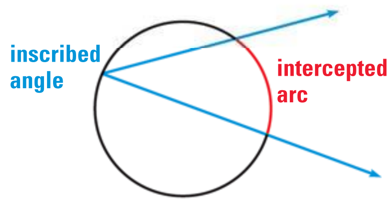
Before You used central angles of circles.

Now You will use inscribed angles of circles.

Why? So you can take a picture from multiple angles, as in Example 4.

Key Vocabulary
- **inscribed angle**
- **intercepted arc**
- **inscribed polygon**
- **circumscribed circle**

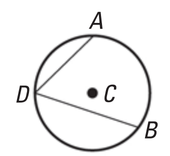
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.



THEOREM — For Your Notebook

THEOREM 10.7 Measure of an Inscribed Angle Theorem

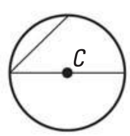
The measure of an inscribed angle is one half the measure of its intercepted arc.



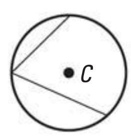
$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

Proof: Exs. 31–33, p. 678

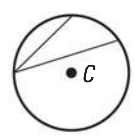
The proof of Theorem 10.7 in Exercises 31–33 involves three cases.



Case 1 Center C is on a side of the inscribed angle.



Case 2 Center C is inside the inscribed angle.

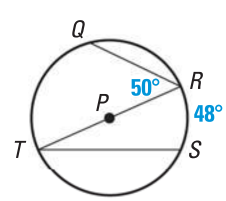


Case 3 Center C is outside the inscribed angle.

EXAMPLE 1 Use inscribed angles

Find the indicated measure in $\odot P$.

a. $m\angle T$ **b.** $m\overset{\frown}{QR}$



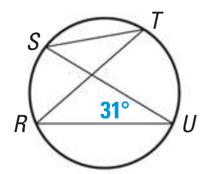
Solution

a. $m\angle T = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(48°) = 24°$

b. $m\overset{\frown}{TQ} = 2m\angle R = 2 \cdot 50° = 100°$. Because $\overset{\frown}{TQR}$ is a semicircle, $m\overset{\frown}{QR} = 180° - m\overset{\frown}{TQ} = 180° - 100° = 80°$. So, $m\overset{\frown}{QR} = 80°$.

EXAMPLE 2 Find the measure of an intercepted arc

Find $m\overset{\frown}{RS}$ and $m\angle STR$. What do you notice about $\angle STR$ and $\angle RUS$?



Solution

From Theorem 10.7, you know that $m\overset{\frown}{RS} = 2m\angle RUS = 2(31°) = 62°$.

Also, $m\angle STR = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(62°) = 31°$. So, $\angle STR \cong \angle RUS$.

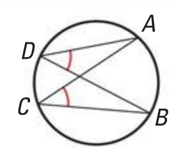
INTERCEPTING THE SAME ARC Example 2 suggests Theorem 10.8.

THEOREM *For Your Notebook*

THEOREM 10.8

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

Proof: Ex. 34, p. 678



$\angle ADB \cong \angle ACB$

EXAMPLE 3 Standardized Test Practice

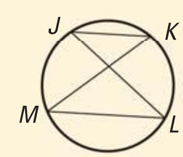
Name two pairs of congruent angles in the figure.

Ⓐ $\angle JKM \cong \angle KJL$, $\angle JLM \cong \angle KML$

Ⓑ $\angle JLM \cong \angle KJL$, $\angle JKM \cong \angle KML$

Ⓒ $\angle JKM \cong \angle JLM$, $\angle KJL \cong \angle KML$

Ⓓ $\angle JLM \cong \angle KJL$, $\angle JLM \cong \angle JKM$



ELIMINATE CHOICES
You can eliminate choices A and B, because they do not include the pair $\angle JKM \cong \angle JLM$.

Solution

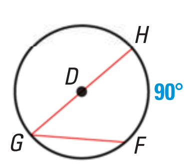
Notice that $\angle JKM$ and $\angle JLM$ intercept the same arc, and so $\angle JKM \cong \angle JLM$ by Theorem 10.8. Also, $\angle KJL$ and $\angle KML$ intercept the same arc, so they must also be congruent. Only choice C contains both pairs of angles.

▶ So, by Theorem 10.8, the correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

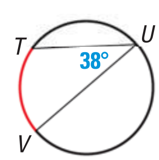
✓ GUIDED PRACTICE for Examples 1, 2, and 3

Find the measure of the red arc or angle.

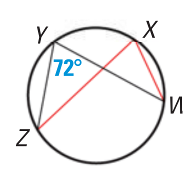
1.



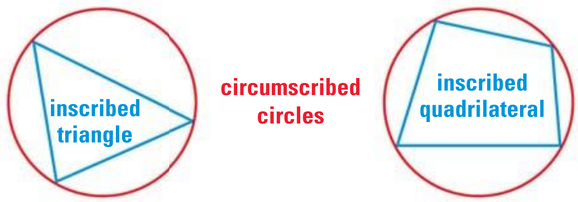
2.



3.



POLYGONS A polygon is an **inscribed polygon** if all of its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.

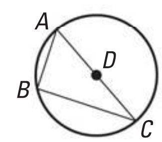


THEOREM

For Your Notebook

THEOREM 10.9

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.



$m\angle ABC = 90°$ **if and only if** $\overline{AC}$ **is a diameter of the circle.**

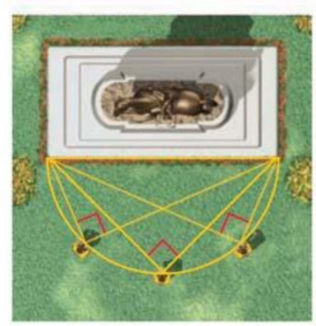
Proof: Ex. 35, p. 678

EXAMPLE 4 Use a circumscribed circle

PHOTOGRAPHY Your camera has a 90° field of vision and you want to photograph the front of a statue. You move to a spot where the statue is the only thing captured in your picture, as shown. You want to change your position. Where else can you stand so that the statue is perfectly framed in this way?

Solution

From Theorem 10.9, you know that if a right triangle is inscribed in a circle, then the hypotenuse of the triangle is a diameter of the circle. So, draw the circle that has the front of the statue as a diameter. The statue fits perfectly within your camera's 90° field of vision from any point on the semicircle in front of the statue.



✓ GUIDED PRACTICE for Example 4

4. **WHAT IF?** In Example 4, *explain* how to find locations if you want to frame the front and left side of the statue in your picture.

INSCRIBED QUADRILATERAL Only certain quadrilaterals can be inscribed in a circle. Theorem 10.10 describes these quadrilaterals.

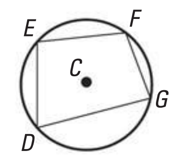
THEOREM — *For Your Notebook*

THEOREM 10.10

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

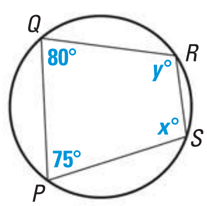
Proof: Ex. 30, p. 678; p. 938



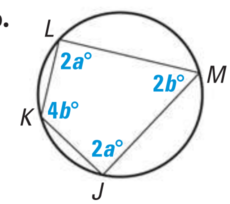
EXAMPLE 5 Use Theorem 10.10

Find the value of each variable.

a.



b.



Solution

a. *PQRS* is inscribed in a circle, so opposite angles are supplementary.

$$m\angle P + m\angle R = 180° \qquad m\angle Q + m\angle S = 180°$$

$$75° + y° = 180° \qquad 80° + x° = 180°$$

$$y = 105 \qquad x = 100$$

b. *JKLM* is inscribed in a circle, so opposite angles are supplementary.

$$m\angle J + m\angle L = 180° \qquad m\angle K + m\angle M = 180°$$

$$2a° + 2a° = 180° \qquad 4b° + 2b° = 180°$$

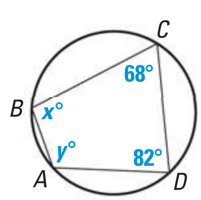
$$4a = 180 \qquad 6b = 180$$

$$a = 45 \qquad b = 30$$

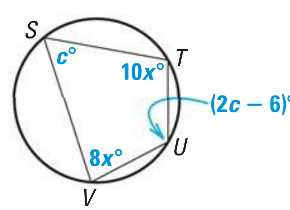
✓ GUIDED PRACTICE for Example 5

Find the value of each variable.

5.



6.



10.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 13, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 18, 29, and 36

SKILL PRACTICE

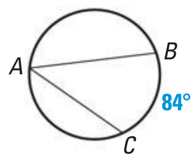
1. VOCABULARY Copy and complete: If a circle is circumscribed about a polygon, then the polygon is _?_ in the circle.

2. ★ WRITING *Explain* why the diagonals of a rectangle inscribed in a circle are diameters of the circle.

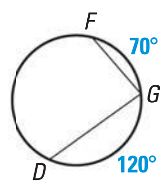
EXAMPLES 1 and 2 on pp. 672–673 for Exs. 3–9

INSCRIBED ANGLES **Find the indicated measure.**

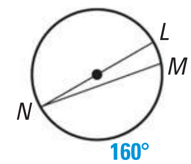
3. $m\angle A$



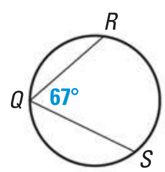
4. $m\angle G$



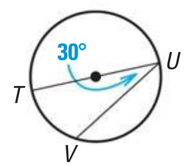
5. $m\angle N$



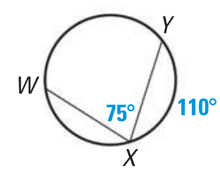
6. $m\widehat{RS}$



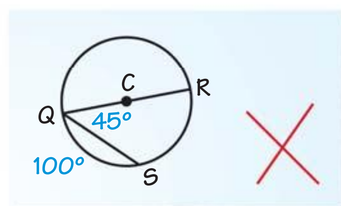
7. $m\widehat{VU}$



8. $m\widehat{WX}$



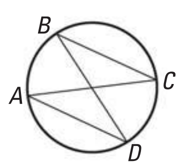
9. ERROR ANALYSIS *Describe* the error in the diagram of $\odot C$. Find two ways to correct the error.



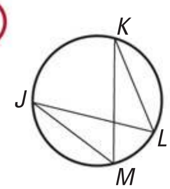
EXAMPLE 3 on p. 673 for Exs. 10–12

CONGRUENT ANGLES **Name two pairs of congruent angles.**

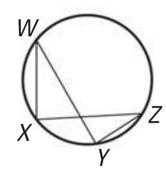
10.



11.



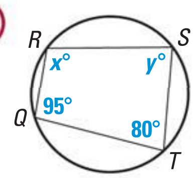
12.



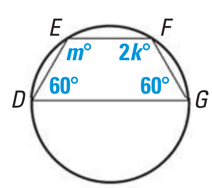
EXAMPLE 5 on p. 675 for Exs. 13–15

ALGEBRA **Find the values of the variables.**

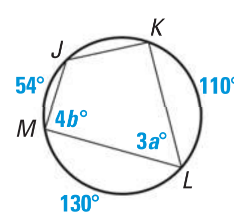
13.



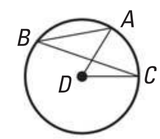
14.



15.



16. ★ **MULTIPLE CHOICE** In the diagram, $\angle ADC$ is a central angle and $m\angle ADC = 60°$. What is $m\angle ABC$?



Ⓐ 15° Ⓑ 30°

Ⓒ 60° Ⓓ 120°

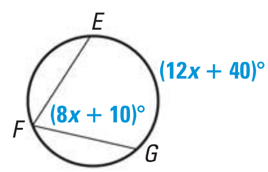
17. **INSCRIBED ANGLES** In each star below, all of the inscribed angles are congruent. Find the measure of an inscribed angle for each star. Then find the sum of all the inscribed angles for each star.

a.

b.

c.

18. ★ **MULTIPLE CHOICE** What is the value of x?



Ⓐ 5 Ⓑ 10

Ⓒ 13 Ⓓ 15

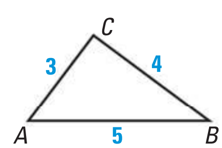
19. **PARALLELOGRAM** Parallelogram $QRST$ is inscribed in $\odot C$. Find $m\angle R$.

REASONING **Determine whether the quadrilateral can always be inscribed in a circle. *Explain* your reasoning.**

20. Square **21.** Rectangle **22.** Parallelogram

23. Kite **24.** Rhombus **25.** Isosceles trapezoid

26. **CHALLENGE** In the diagram, $\angle C$ is a right angle. If you draw the smallest possible circle through C and tangent to $\overline{AB}$, the circle will intersect $\overline{AC}$ at J and $\overline{BC}$ at K. Find the exact length of $\overline{JK}$.



PROBLEM SOLVING

27. **ASTRONOMY** Suppose three moons A, B, and C orbit 100,000 kilometers above the surface of a planet. Suppose $m\angle ABC = 90°$, and the planet is 20,000 kilometers in diameter. Draw a diagram of the situation. How far is moon A from moon C?

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EXAMPLE 4
on p. 674
for Ex. 28

28. **CARPENTER** A *carpenter's square* is an L-shaped tool used to draw right angles. You need to cut a circular piece of wood into two semicircles. How can you use a carpenter's square to draw a diameter on the circular piece of wood?

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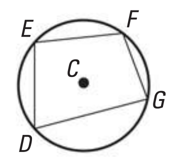
29. ★ **WRITING** A right triangle is inscribed in a circle and the radius of the circle is given. *Explain* how to find the length of the hypotenuse.

30. **PROVING THEOREM 10.10** Copy and complete the proof that opposite angles of an inscribed quadrilateral are supplementary.



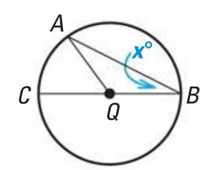
GIVEN ▶ $\odot C$ with inscribed quadrilateral $DEFG$

PROVE ▶ $m\angle D + m\angle F = 180°$, $m\angle E + m\angle G = 180°$.

By the Arc Addition Postulate, $m\overset{\frown}{EFG}$ + __?__ $= 360°$ and $m\overset{\frown}{FGD} + m\overset{\frown}{DEF} = 360°$. Using the __?__ Theorem, $m\overset{\frown}{EDG} = 2m\angle F$, $m\overset{\frown}{EFG} = 2m\angle D$, $m\overset{\frown}{DEF} = 2m\angle G$, and $m\overset{\frown}{FGD} = 2m\angle E$. By the Substitution Property, $2m\angle D$ + __?__ $= 360°$, so __?__ . Similarly, __?__.

PROVING THEOREM 10.7 If an angle is inscribed in $\odot Q$, the center Q can be on a side of the angle, in the interior of the angle, or in the exterior of the angle. In Exercises 31–33, you will prove Theorem 10.7 for each of these cases.

31. **Case 1** Prove Case 1 of Theorem 10.7.

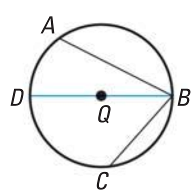


GIVEN ▶ $\angle B$ is inscribed in $\odot Q$. Let $m\angle B = x°$. Point Q lies on $\overline{BC}$.

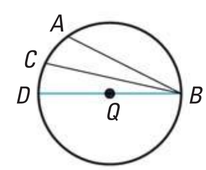
PROVE ▶ $m\angle B = \frac{1}{2}m\overset{\frown}{AC}$

Plan for Proof Show that $\triangle AQB$ is isosceles. Use the Base Angles Theorem and the Exterior Angles Theorem to show that $m\angle AQC = 2x°$. Then, show that $m\overset{\frown}{AC} = 2x°$. Solve for x, and show that $m\angle B = \frac{1}{2}m\overset{\frown}{AC}$.

32. **Case 2** Use the diagram and auxiliary line to write GIVEN and PROVE statements for Case 2 of Theorem 10.7. Then write a plan for proof.



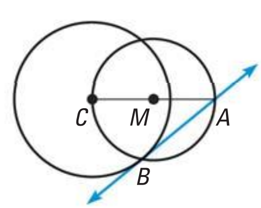
33. **Case 3** Use the diagram and auxiliary line to write GIVEN and PROVE statements for Case 3 of Theorem 10.7. Then write a plan for proof.



34. **PROVING THEOREM 10.8** Write a paragraph proof of Theorem 10.8. First draw a diagram and write GIVEN and PROVE statements.

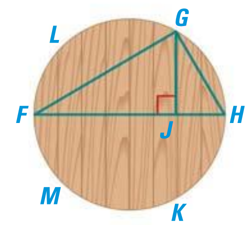
35. **PROVING THEOREM 10.9** Theorem 10.9 is written as a conditional statement and its converse. Write a plan for proof of each statement.

36. ★ **EXTENDED RESPONSE** In the diagram, $\odot C$ and $\odot M$ intersect at B, and $\overline{AC}$ is a diameter of $\odot M$. *Explain* why $\overleftrightarrow{AB}$ is tangent to $\odot C$.



CHALLENGE **In Exercises 37 and 38, use the following information.**

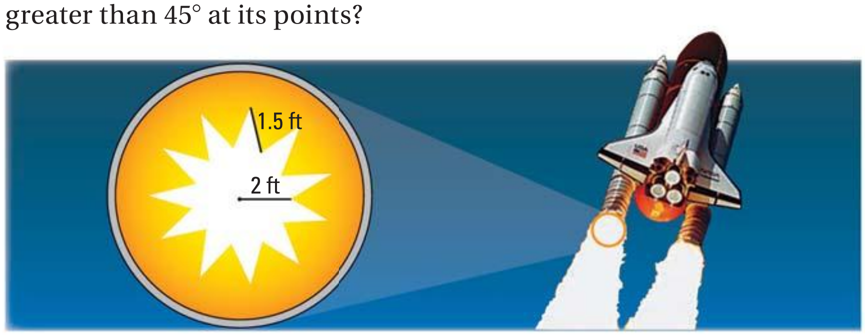
You are making a circular cutting board. To begin, you glue eight 1 inch by 2 inch boards together, as shown at the right. Then you draw and cut a circle with an 8 inch diameter from the boards.



37. $\overline{FH}$ is a diameter of the circular cutting board. Write a proportion relating GJ and JH. State a theorem to justify your answer.

38. Find FJ, JH, and JG. What is the length of the cutting board seam labeled $\overline{GK}$?

39. **SPACE SHUTTLE** To maximize thrust on a NASA space shuttle, engineers drill an 11-point star out of the solid fuel that fills each booster. They begin by drilling a hole with radius 2 feet, and they would like each side of the star to be 1.5 feet. Is this possible if the fuel cannot have angles greater than 45° at its points?

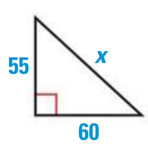


MIXED REVIEW

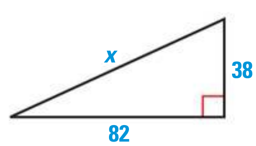
PREVIEW
Prepare for Lesson 10.5 in Exs. 40–42.

Find the approximate length of the hypotenuse. Round your answer to the nearest tenth. *(p. 433)*

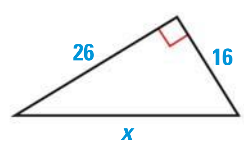
40.



41.

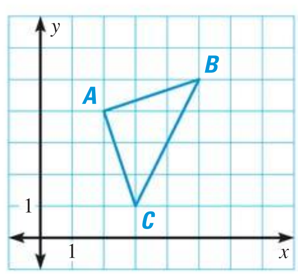


42.

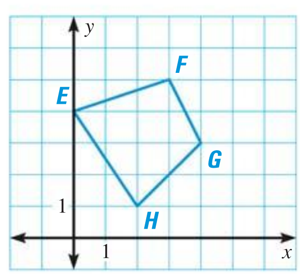


Graph the reflection of the polygon in the given line. *(p. 589)*

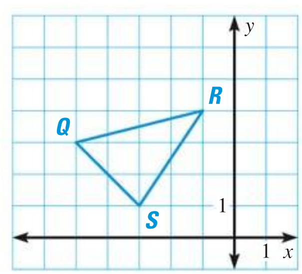
43. y-axis



44. $x = 3$



45. $y = 2$



Sketch the image of $A(3, -4)$ after the described glide reflection. *(p. 608)*

46. **Translation:** $(x, y) \to (x, y - 2)$
Reflection: in the y-axis

47. **Translation:** $(x, y) \to (x + 1, y + 4)$
Reflection: in $y = 4x$